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An Approach to the Modelling of Urban Housing Markets

Part 1: Housing Demand

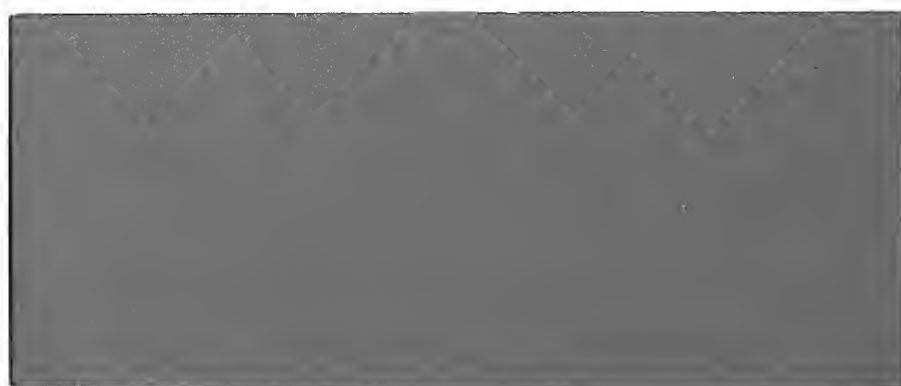
Jerome Rothenberg

No. 149

February, 1975

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I Distinctive Characteristics of Urban Housing Markets

Urban housing markets are not a good representative of the abstract general market of conventional economic theory. They possess a number of distinctive features that are, at one and the same time, integral to urban housing and difficult to integrate within conventional market theory. Inclusion of these features does considerably complicate the analytic, and even more the econometric, treatment of the field; but omission in the interests of tractability may seriously misrepresent the phenomena being modeled. The purpose of this essay is to bring to attention some of these distinctive features, and to delineate a theoretical approach which attempts to grapple with them in a constructive manner. The approach underlies a particular set of currently ongoing econometric studies in which the author is involved, but is compatible with a broader family of empirical efforts.

The two most important features of urban housing are the localization in consuming housing services and the durability of housing capital.

1. Localization. Housing is a consumption activity that for an overwhelming proportion of households occurs at a single spatial location during each conventional economic period (say one year). Commodities are produced in a great variety of places and are transported to—or by—consumers for consumption at one or more places, or consumers selectively travel to various sites where different instances of consumption may occur. Housing does not possess either the portability of this typical commodity or the variety of sites at which sequences of consumption may take place. For most households consumption of housing services takes place at a single site, where the

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selected combination of components of the housing package exists. Housing therefore serves as an anchoring function, "placing" a household at a particular spatial location which, because of the real costs in overcoming distance, becomes a base for a variety of other forms of consumption involving both private and public goods. It is this localization that creates the concept of residential neighborhood and household accessibility, and hence deeply links the economic dimension of urban activities with the sociological and political dimensions.

2. Durability

Housing capital is by far the most durable capital incorporated in consumer goods. Indeed, it is much more durable than any form of business capital as well, if the longevity of housing units in older countries like Europe is considered -- units at every quality level which are up to four and five hundred years old!

Durability means more than simply the continued physical existence of a structure. It means that a structure does not ineradicably lose its current marketability just because of growing age. An older unit remains a good substitute in the market for units just built, even at advanced age. This continued relative serviceability is due partly to the fact that technological change in residential construction has not so much changed the nature of housing services as the materials used to produce them. The more fundamental changes in the nature (or quality) of the services have generally been associated with capacities installable in older units at modest cost or with equipment obtainable independently of the housing unit proper.

Not only does economic viability remain with age, it is not marginal. While depreciation does occur with time and usage, maintenance, repair and larger improvements can enable high quality units to continue to offer high quality services through

advancing age. Thus, old houses are not necessarily poor houses. They can exist throughout the spectrum of quality. Advancing age does influence the cost at which any given quality can be maintained: it becomes more expensive with time. Thus, via an economic calculation, not as physical necessity, older houses are often observed to be of low quality (or in poor condition); but in other market circumstances the higher maintenance costs are worth making, and units are observed at higher qualities (and better conditions).

We have stressed the persistence of economic viability in terms of the services that older units render. But there is another side to durability. Housing structures are very expensive to do away with. Even where age makes serviceability disappear it does not make the structure disappear. Units which are desired to be retired may simply be allowed to crumble. But the site occupied by them cannot be reclaimed for re-use unless the structures are physically demolished. This demolition is very expensive, amounting to a not inconsiderable proportion of the original cost of the structure itself (at constant prices). Thus, even when not wanted, old structures tend to remain in existence and, under many circumstances, this continued existence makes it economically worthwhile to continue to use them for housing.¹

3. Multi-dimensional Heterogeneity

Housing units in an urban area are heterogeneous, not homogeneous. They differ markedly in a number of dimensions. Differences in the physical units themselves are

¹The recent substantial growth in abandonments shows an opposite configuration. Rapid neighborhood decline due to demographic factors, tax considerations, public regulations like rent control, make for a sudden collapse of profitability not associated with age. These units are withdrawn from the housing market despite basic structural integrity.

well-known, embracing size, number of rooms, architectural layout and style, structural amenities, and condition, among others. But a housing unit comprises more than simply the structure, because of the localization function referred to above. It is a package, consisting of the structure, the land lot, the neighborhood and the configuration of accessibility to different desirable destinations within the urban area and outside. Each of these other dimensions displays wide differences as well. Lots differ in size, topography, view, placement on the block, etc. Neighborhoods differ in demographic character, assortment of private goods available, quality and variety of public goods provided, density and land-use mix. Locations differ greatly in the real cost involved in reaching different desirable destinations because of distance, and availability and quality of different transportation modes.

Most commodities can be shown to be packages of several components, and some or all of these components can be shown to differ for different brands or sellers. But heterogeneity has some special features for urban housing. First, households differ substantially in their tastes for housing. They differ in the importance they ascribe to alternatives on each dimension of the package, and in the relative importance of the different dimensions -- i.e., the preference tradeoffs within each component and across components. This means that the differences to be observed in actual units matter in the market, and are thus likely not to be accidental but at least to a degree intended.

Second, households behave as though these differences matter a great deal. They engage in very substantial (and thus costly) search in order to find an especially appropriate package. Also, they show a willingness to pay very high premiums for notably attractive combinations. Price differences in the market are considerable for packages that seem to differ only slightly.

Third, while for most commodities the multiple dimensions that matter to consumers are subject to the control of producers and sellers, this is significantly untrue for urban housing. A producer/owner of a housing unit can substantially control the structure characteristics of the unit over time and, to a lesser extent, the land component. But the neighborhood and accessibility components are at most selected by the producer/owner at the outset (and even this selection is likely to be severely constrained), and all subsequent changes are outside his control. Since such subsequent changes can have large effects on the overall desirability of the package, and since they occur without the direct mediation of market transactions between the owner and those responsible for the changes, their existence amounts to a serious potential for pervasive externalities within urban housing markets.

4. Convertibility of Existing Units

Housing units are highly durable. But their durability is not independent of human action during the aging process. Continued viability in the market requires continued maintenance and repair. Indeed, human intervention during the lifetime of a housing unit can have more radical impact than ensuring its sheer economic survival. A unit constructed to render services at a certain quality level can be converted during its lifetime to render services at either a higher or lower level. Upward conversion is accomplished by making additions to a unit, or improvement of plumbing or electricity, or of upkeep, making architectural changes, or consolidating two or more originally separate units into a single larger unit. In most of these, real resources are being invested to create an improvement.

Downward conversion has two main forms. One is like upward conversion, involving

an investment of real resources to decrease the quality of individual units. This occurs where one unit is split into two or more units, each one containing less space and probably fewer rooms than its predecessor. Investment is required here to put up additional partitions and provide extra utilities to make each unit complete. The second involves decreasing the level of maintenance and repair so that usage and time will gradually worsen the condition of the unit.

5. Capital and Current Transactions: The Capital Market

In urban America, given conventional standards of living and their translation into technology, a housing unit is a very expensive commodity. Most people occupy units whose market value is at least three times as great as their annual income and greater than their total portfolio of assets. The durability of housing makes it possible to accommodate to the budget and wealth position of households by two means: 1) high mortgages on ownership claims; 2) a very important rental market. Probably a larger proportion of housing services are rented than for any other consumer good. Both of these market patterns lead the housing sector to make very heavy demands on loanable funds, especially long-term funds. Housing is one of the largest clients of the national capital market.

6. Moving Costs

To a much greater extent than with other types of consumer goods, a change in consumption of housing is costly. First, since heterogeneity in the available stock of housing is considerable, and tastes regarding these differences quite important, substantial search may be required for a household to assure itself of a satisfactory change. Second, even after search is ended, the transaction itself may be costly, substantial

legal and other specialized services are required when a sales transaction occurs, and may even be necessary when long-term lease agreements are involved. The situation is especially complicated when an owner-occupier household wishes to buy a different house, because then the household must not only incur the costs of finding and buying, but must also sell its present unit. Third, a move to new dwelling entails the cost of physically moving a household's possessions, an expensive and often time-consuming process.

One last aspect of moving costs stems from the localization function performed by housing. A household will often imbue its present neighborhood with an emotional aura because of the presence there of close friends and relatives, and because of past associations. This is lost to a greater or lesser extent when moving, depending on which move is involved. For some families this can be an extremely heavy cost. It is not so invariably, however, since families differ in their attitudes toward their neighborhoods. Some actually dislike theirs, and it is in fact the desire to move out of the neighborhood that induces them to change housing. Despite differences, the disruption of a particular style of life anchored around housing is a potentially important element in considering the cost of changing housing.

The above items all refer to a change in consumption brought about by physically changing the site of housing consumption. But an alternative way to change without moving is to convert a household's present residence to provide the desired changes in housing services. This is generally not possible with rental occupancies, and even with ownership occupancies it is sometimes constrained by the existing lot and perhaps even by zoning. Moreover, it applies only to structure services and not to other aspects of the housing package. It is, however, a substitute of variable attractiveness for

physical moves that avoids the substantial costs of moving.

7. Public constraints

Because of the localization function, and the close association of physical housing conditions on health and safety, housing has become the target of a variety of public interventions, including zoning regulations, health and building codes, and rent control, among others. These can exercise substantial constraints on the actions of suppliers and demanders in the market, and on the working of the market itself. The housing market is not a perfectly free market.

II Some Operational Consequences

1. Pre-existing Stock and New Units

Because of the durability of housing, units of many ages will offer services at any time. Indeed, in any year more than ninety percent of all units offering services will have been built before the beginning of the year. The pre-existing stock is the paramount component in the supply of housing in any period. Newly produced units are a small part of the story, although they may have influence disproportionate to their numbers in some respects.

The pre-existing stock is not, however, a passive component of the supply of housing. Because of convertibility,² owners of existing units can modify them in directions suggested by changing market opportunities. Thus there are two forms of active supply response to market forces: the building of new units and the conversion of older units. While the two have similarities, they have differences as well. The

²Similar to one of the several, sometimes confusing, meanings of "filtering" in the housing literature.

sheer magnitude of the preexisting stock makes the conversion form of supply eminently deserving of understanding.

2. Information

The great variety of housing packages and the importance of tastes regarding differences in those packages lends corresponding importance to information. Active participants in the market must be adequately informed about alternatives to avoid large opportunity losses. Households looking for a unit can expect a wide variety to be available. Substantial search is called for, often lengthy face-to-face search because of the multiple dimensions to be examined.

Costly information is important to sellers as well. The very same diversity in the market makes it difficult for a prospective seller to know the most favorable terms he can realize for his particular unit. He knows that prospective buyers will differ in tastes and information, and therefore in what they are willing to offer for his unit. He must therefore "search" among prospective buyers for the most suitable. His "search" consists in deciding on asking prices, willingness to bargain, length of time to wait before bargaining, etc. This search, no less than buyer search, involves cost--opportunity costs here, as against the combination of opportunity cost and active search costs for buyers.

3. Market Segmentation

The great diversity of housing packages in the market means that they are not all perfect or even near-perfect substitutes for one another. They are likely to exhibit a whole spectrum of substitutive relationships, from very close to nearly non-existent. It is even possible that various irregularities may exist--e.g., where households differ

markedly in the degree of substitutability they accord within a given set of units.

The presence of this spectrum of substitutability means that "the" urban housing market is not one market but a complex of differentially related sub-markets. Each sub-market is a cluster of units widely considered close substitutes, and related to other clusters in terms of the differing degrees of substitutability with them.

This kind of segmentation of the market is abetted by the high cost of adequate information for participants. Participants cannot afford to scan all or even a major part of the market. They delimit their search to the most "relevant" portion of the market, a demarcation probably based on various rules of thumb that reflect prevailing knowledge about highly substitutive clusters. The deliberate decision to avoid being well-informed outside modest portions of the market would tend to consolidate both the inequality of substitutability and the unevenness in the distribution of degree.

One consequence of this segmentation is that the different submarkets can experience an independent margin of variation relative to one another: one can have high excess demand, another excess supply, another be well-balanced. So housing prices may move disparately among them. Another consequence is that variation initiated within one subsector will have very uneven repercussions on the other subsectors: in general, the closer the relatedness the more nearly parallel the repercussion in direction and intensity.

4. Transactional Friction

The high costs of active participation in the market-- search, transactions and moving costs -- imply that the prospective gains from becoming an active participant (i.e., seeking to make a change) may have to be quite substantial to warrant such participation. Prospective gains arise from changes within the household that affect its housing preferences (among different combinations of the components of the housing

package, and between housing and non-housing commodities), and from changes in the character and prices of different packages available on the market. Both of these are likely to occur gradually most of the time, although some sudden changes in both do happen.

This suggests that a household that has just made a change is not likely to make another change very soon because of the high participation costs. So active participation is not likely to be continuous for most households but sporadic, with long periods of non-participation not uncommon. This in turn implies that in such period the particular lineup of units available for sale or rental and households seeking units is very important in determining the outcomes of current transactions. Some adaptive self-selection may be involved in determining this lineup, but chance factors are likely to be present as well. The upshot of this raises real questions about the overall efficiency of the market in pairing wants with availabilities, both at any one time and when adaptive responses are made over time by both demanders and suppliers (in the latter because they are adapting to signals that are distorted).

5. Neighborhood Externalities

The localization function of housing makes the neighborhood component of the housing package important. Since third party changes in neighborhood can affect the desirability of a housing unit independently of the actions of its owner or taste changes by its user, external effects in the housing market can be strong and pervasive. The usual market distortions resulting from externalities are likely to result. In addition, less usual forms of resource immobility can result. For example, adaptation to new market opportunities might be optimal if a group undertook it, yet no one member of the group finds it worthwhile to undertake without the expectation that the others will do so too.

Decentralized behavior fails to secure such an expectation and so results in inappropriate individual--and hence aggregate--market performance.

6. Vulnerability to Capital Market

The heavy dependence on the national capital market gives the housing sector a special vulnerability. Aggregate forces can impinge on the capital market for reasons that have little to do with the worth-whileness of housing versus other production or consumption sectors; yet these may have a powerful positive or negative impact on housing. Macroeconomic conditions may call for heavy use of monetary policy. This will have effects in housing for greater than on other production and consumption sectors, yet unmotivated by any real change in relative national priorities among the sectors.

Thus, vulnerability to the national capital market makes for rather special features in housing.

7. Dynamics

Most of the foregoing suggests that actual sequences and timings of actions by participants matter. Disequilibria maintained through frictions and, uninformedness and immobilities, and substantial adjustment lags, make it important to understand ongoing processes, even more, perhaps, than hypothetical equilibrium destinations. Indeed, they even raise the question whether equilibrium is ever to be observed under any but very special circumstances. The housing market's most distinctive features may well be dynamic, and so efforts to understand it theoretically may have to have much the character of dynamic models.

In what follows I shall present a dynamic model of urban housing markets. The

present paper includes the portion devoted to housing demand. Papers being currently completed deal with supply and market adjustment. The overall model explicitly attempts to embed many of the distinctive characteristics of urban housing, and to comprehend many of the asserted consequences described above. On the other hand, it is by no means being claimed that this model definitively handles most of the inherent complexities of the reality being modeled. Indeed, it is not even fully developed within its own presumed scope. It is being presented as one of a series of theoretical studies dealing with a common approach to the understanding of urban housing phenomena.

III A Dynamic Model of Urban Housing Markets

A. Individual Short-Run Housing Demand

1. Utility Function

We begin with an urban population containing N households, some similar but at least some different. Each household has a utility function, which may be similar to or different from, that of others.

$$(1) U^i = U^i(Z, H) \quad i = 1, \dots, N$$

where Z is the composite service flow of all other commodities (the numeraire commodity); H is the amount or quality of housing services rendered in the present period and is in fact a vector of housing package components, each itself a vector:

$$(2) H = (H_S, H_L, H_N, H_A, H_T)$$

where H_S is the vector of housing structure characteristics like number of rooms, interior size, architectural style, interior amenities (number of bath rooms, fireplace, heating system, etc.), and condition;

H_L is the vector of lot characteristics, like size, shape, placement in block, topography, etc.;

H_N is the vector of neighborhood characteristics, like demography, variety of private goods available, character of public services rendered (sanitation, schools, parks, etc.), aesthetic character, land use mix, etc.

H_A is the vector of accessibility to the various desirable destinations within and outside of the urban area.

H_T is the tenure status; it reflects a style of occupancy; in addition, it proxies for certain idiosyncratic features of a unit, those reflecting the often custom-made character of ownership units as opposed to the more usually standardized character of rental units (appropriately designed for a larger number of occupants with a variety of tastes).

The shape of U^i with respect to the housing components -- tradeoffs among the H_i and H and Z -- depends on socioeconomic characteristics of household i , F^i :

$$(3) F^i = [Y_i, N_{1i}, N_{2i}, A_i, E_i, R_i, W_i, \dots] \quad i = 1, \dots, N$$

where Y_i is the permanent income level of household i

N_{1i} is the household size

N_{2i} is the number of school age children

A_i is the age of the household head

R_i is a designation of ethnic group or race

W_i is the amount of liquid assets owned

2. Housing Package of Components and Component Prices

The five types of components listed in equation (2) are not purchased separately by the consumer, but as indissoluble parts of the overall package. Relative to some starting package, if one wants to change one or more components one has to select

a different overall package rather than simply transforming each while holding the rest unchanged in the same package.³ If the housing market is large and varied enough, a consumer is faced with a very broad range of different component combinations. In effect, he then can put together almost whatever combination he wants--as though he were in fact buying each component separately.

This suggests that the several housing components could be treated like separate commodities, and the consumer's budget allocation could then be treated as an allocation over the numeraire commodity Z and the various housing components rather than over something called housing. Which is more appropriate? The problem is salient because while the components are parts of packages, we have not spoken yet about a magnitude of housing packages. Housing is a vector. Strictly speaking, we can speak unambiguously of size of the package only if all components increase or decrease together, or less generally, if some components change in one direction (greater or lesser) and none in the other. But we consider some pairs of components to be substitutes, some to be complements and some--under certain circumstances--to be sometimes one, sometimes the other. So we expect to find consumers voluntarily increasing consumption of one component while decreasing that of another, and we must be prepared to infer something about the "size" of different packages where vector dominance does not hold. This is quite difficult, but it turns out that both approaches involve important difficulties, yet the consumer process can be viewed in both ways.

We begin by treating consumer choice over components as separate commodities.

³A class of exceptions was noted above with respect to structure components. Some desired structure changes can be physically accomplished by conversion, thereby holding all non-structure components unchanged in the original package.

To develop this analogy we need to be able to designate separate prices for the separate component commodities. This has two parts. First, we must be able to quantify each component so as to be able to distinguish--at the very least--between more or less of the component. Second, we must be able to specify a unit of quantity, so that a price per unit may be specified; or, failing that, we must be able to distinguish different states of the component (a nominal scale) so that a different overall price can be ascribed to each state.

The first requirement is not trivial, since all the components are vectors, most are vectors of vectors, and whereas some vector dimensions lend themselves easily to choice-relevant quantification, others do not. We start by assuming that market participants can agree on the direction of more or less, or more or less preferred for each dimension. Even this fails for some dimensions and sub-dimensions. Architectural style, certain amenities, will mean quite different ordinal things to different people. With such profoundly judgmental dimensions we can develop at best nominal scales, abandoning the concept of an intersubjective ordering scale.

Since each component is a vector, we must either treat the ultimate separate commodities as the most proximate set of scalar dimensions, or attempt to collapse each constituent set of sub-dimensions to a scalar index representing each first level component (i.e. H_S, H_L, H_N, H_A, H_T). Since the problem of indexing these dimensions is formally very similar to that involved in the concept of the "amount of housing" we leave it to the later discussion. We therefore treat a new set of components as the commodities entering the consumer's utility function, namely, the scalar constituents of the elements of (2):

$$(4) H = \{H_1, H_2, \dots, H_k, \dots, H_M\}$$

What are the corresponding prices of the "commodities"? Since they are not in fact bought separately, no separate prices are quoted for them. But each package of components has a quoted price, and these prices are in effect an aggregate of the implicit prices of the several constituents of the package. Adopting the assumption that package prices are linear aggregates, the relation between package and component prices for a given package is⁴:

$$(5) \quad P_H = \sum_{k=1}^M P_k H_k$$

where P_H is the market price of the housing package;

P_k is the implicit market price of component H_k ;

H_k is the amount of component k embodied in the package.

Since the consumer in the market is faced with a large set of obtainable packages, he receives the information reflected in the matrix equation:

$$(6) \quad P_H = H P$$

where P_H is the $Q \times 1$ column vector of market package prices

H is the $Q \times M$ matrix:

$$\begin{bmatrix} H_{11} & H_{12} & \dots & H_{1M} \\ H_{21} & \dots & \dots & H_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ H_{Q1} & \dots & \dots & H_{QM} \end{bmatrix}$$

so that row i is the constituent characteristics of package i

P is the $M \times 1$ column vector of implicit constituent prices.

This is a set of simultaneous equations that, assuming fulfillment of the formal requirements for a solution is given by:

$$(7) \quad H^{-1} P_H = P$$

⁴Since our commodities are annual service flows, prices here refer to rental costs, whether explicit or -- for owner-occupiers-- implicit.

Thus, market observations of package prices contain enough information in principle to enable one to discover the implicit prices associated with the package components-- if all sellers are implicitly quoting the same price for each component. We assume that this last condition is met by the competitive forces operating in the market. If in an early iteration of asking prices one seller is implicitly asking a higher price for a given component than are other sellers an early impressionistic set of calculations by buyers will disclose the overpricing and the seller will fail to sell his unit unless he adjusts the outlying component price. Thus, market clearing processes and calculation of component prices occur effectively simultaneously.

We therefore define a set of implicit component prices at each time t . With these prices we specify the budget constraint facing household i :

$$(8) \quad Y^i = \sum_{k=1}^M P_k H_k + P_z Z \quad (\text{with } P_z = 1)$$

3. Individual Consumer Equilibrium

The consumer's equilibrium at time t is thus given by the solution of the following maximization problem:

$$(9) \quad \max U^i = U^i [Z, H_1, H_2, \dots, H_M | F_t^i]$$

$$\text{subject to } Y_t^i = \sum_{k=1}^M P_{kt} H_k + Z$$

where F_t^i is household i 's socio-economic state at t and P_1, \dots, P_M are given in the market.

This solution, with familiar optimal conditions, shows the optimal combination of housing components and numeraire commodity that should be purchased to maximize utility.

The solution in these terms lacks some intuitive thrust: it does not tell "how much" housing is being purchased. Is it possible to formulate the consumer's choice process in terms of "housing" as a commodity? To do so, we employ an analogy with production

theory. We treat "housing" as the output that results from combining a number of productive inputs--the housing package components. The inputs are related to the outputs by way of a housing production function. This function is in reality a way of speaking about the household's utility function, since there is no objective entity recognizable as "housing" but only a subjective valuation of the overall attractiveness of the complex of services that is conventionally labeled "housing." Therefore there are as many different housing production functions as there are different household utility functions.

By treating housing as the output of a housing production process we can distinguish two aspects or stages of the household's choice problem: (1) the optimal combination of inputs to produce any level of housing output, (2) the optimal level of housing output. Housing level is defined as a given level of utility, for specified consumption of Z and various combinations of housing inputs. We can define housing isoquants as utility level contours, and the family of isoquants map out preference tradeoffs between inputs and between inputs and outputs.

The household's choice problem can be reformulated as follows. Define a set of housing isoquants as:

$$(10) h_w = \left\{ g H^i \mid \Omega_w = U^i [Z, (g H^i) \mid F^i] \right\} \quad \text{all } \Omega_w \text{ in the range of } U^i$$

Thus, each isoquant is the set $g H^i$ of housing components that, for given F^i and Z , give household i utility level Ω_w . The household's maximization problem is now given as:

$$(11) \max U^i = W^i [Z, h^i \mid F^i_t]$$

$$\text{subject to } Y^i_t = p (h^i) h^i + Z$$

where h^i is the level of housing from the point of view of household i ;

P is the price per unit of housing;

$p(h^i)h^i$ is the least cost function for consuming each h^i level of housing, analogous to the firm's total cost function.

In maximizing utility, the household selects the same budgetary allocation as under equation (9). But two conditions for optimality can be expressed in terms of the concepts of equation (11):

$$(12a) \quad h^i_i/p_i = h^i_k/p_k = \dots = \partial h^i / \partial [p(h^i)h^i] \quad \text{or} \quad h^i_i/h^i_k = p_i/p_k \quad \text{all } i, k$$

where $h^i_i = \partial h^i / \partial H_i$ and $h^i_k = \partial h^i / \partial H_k$

$$(12b) \quad [\partial u^i / \partial h^i] / [\partial u^i / \partial Z] = [\partial [p(h^i)h^i]] / \partial h^i$$

(12a) refers to equimarginal net advantage as determining the optimal composition of the housing package and simultaneously the marginal cost of housing; (12b) refers to the optimal level of housing as determined by the marginal utility and marginal cost of housing.

In operationalizing this theory for empirical estimation, the second form of (12a) will be important, since this will contribute toward the actual measurement of housing levels.

4. The Action Situation of the Household

At time t , household i , given Y^i_t and $P_{1t}, P_{2t}, \dots, P_{Mt}$, has determined an optimal pattern of expenditure on housing and all other commodities. Call these optimal magnitudes $\hat{Z}, \hat{H}_1, \hat{H}_2, \dots, \hat{H}_M$ and thus \hat{h}^i as well: or simply (\hat{Z}, \hat{H}) . At the same time

however, household i is already consuming a housing package H_t , and with it, Z_t .

So we can specify a degree of discrepancy between ideal and actual budget pattern:

$\hat{H} - H_t, \hat{Z} - Z_t$. This carries with it a discrepancy between attainable (in principle) utility level and actual utility level. We can refer to these as losses, and measure them in consumer surplus terms.

We can distinguish two dimensions of loss, corresponding to our formulation of the consumer's maximizing activity as relating to optimal composition of housing and optimal level. L_1 is the loss resulting from an improper internal composition; L_2 the loss from improper allocation between housing and all other commodities. The total loss is $L_1 + L_2$.

a. Loss from Improper Composition

This is measured from the baseline established by condition (12a): we presume that (12a) is violated, and measure the extent of the violation:

$$(13) \quad L_1 = 1/2 \sum_i (\hat{H}_i - H_{it}) (h_i^i [\partial p(h^i) h^i] / \partial h^i |_{h^i = \hat{h}^i} - p_i)$$

This is a version of the welfare triangle surplus measure: $1/2 \triangle P \triangle Q$ -- i.e.,

the multiple of wrong price discrepancy and wrong quantity discrepancy. In (13),

$\hat{H}_i - H_{it}$ measures the extent of wrong usage of component i ; the next term measures the extent of wrong price discrepancy, in that fulfillment of (12a) would make the value of marginal product of component i , measured at the optimal level \hat{h}^i , equal to the price of i . The marginal physical product is h_i^i , the value of each extra unit of housing

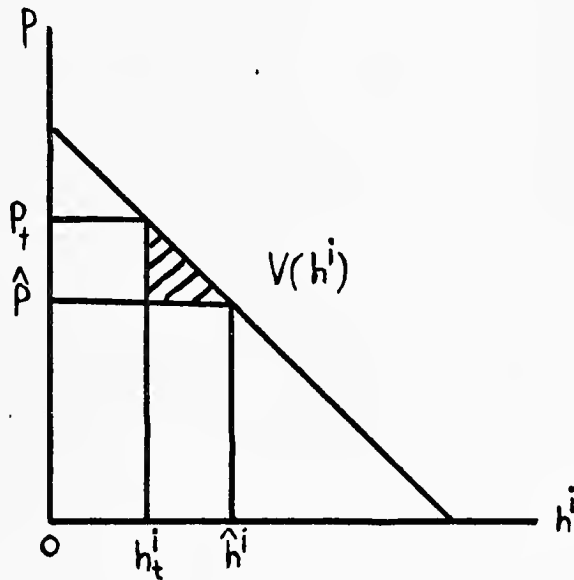
$\partial p(h^i) \hat{h}^i / \partial h^i |_{h^i = \hat{h}^i}$ since in the optimal situation this is shown in (12b) to

equal the relative marginal utility of an extra unit of housing. In effect, (13) measures something like the extra cost of consuming housing when the internal composition is not the least cost composition as defined by an optimal choice.

b. Loss from Improper Level

This implies violation of (12b). We measure the cost of violation as a sum of welfare triangles under demand (or valuation) curves. See Figure 1.

Figure 1



$V(h^i)$ a valuation function: $p = f(h^i)$. The shaded triangle is the loss from operating at (h_t^i, p_t) rather than at (h^i, \hat{p}) . It is given by:

$$(14) \quad L_2 = 1/2 [|h_t^i - \hat{h}^i| \cdot |V(h_t^i) - V(\hat{h}^i)|] + 1/2 [|Z_t - \hat{Z}| \cdot |Z_{V(Z_t)} - Z_{V(\hat{Z})}|]$$

The terms referring to Z indicate that the discrepancy in consumption of housing has created a corresponding discrepancy in the consumption of Z , so the losses in both markets must be registered.

c. Total Loss

Both L_1 and L_2 refer to flow losses in each time period. It is assumed that they will be repeated in each time period that conditions remain unchanged. Since these losses are to be compared with prospective costs of entering the market to decrease or wipe out these losses, and such costs have a different time shape, we must convert their expected sequence into present discounted value terms: L_1^* and L_2^* respectively.

Then total costs are given by:

$$(15) L^* = L_1^* + L_2^*.$$

When faced with a welfare loss of L^* the household has the option of becoming an active participant in the housing market with the purpose of moving to a more appropriate housing unit-- one that conforms more nearly to \hat{H} . There is, however, a cost associated with active participation. This cost has three components: a cost of search, a cost of conducting a market transaction and a cost of moving to the newly chosen unit. Since the last two are not a function of the amount of search carried on, they can be collapsed together as an "overhead" type of cost of concluding a transaction and changing residence accordingly. Its variability depends on the size of the transaction, the cost of conducting capital transactions, and the amount of property that must be moved.

Thus, we define a transaction and moving cost:

$$(16) C_T = C_T (K^i, r^*)$$

where the cost of conducting capital transaction and the size of the transaction, as well as the amount of property to be moved are made functions of the wealth position of household i , K^i , and either a market rate of interest or a borrowing minus lending rate differential, r^* .

The other cost of active participation is the cost of search. Search cost depends on the amount of search conducted, and is a rising function of that amount. The search decision can be looked at in the following way. \hat{H} represents an ideal housing configuration for household i . But it has not been found concretely in the market; it is only a goal of the search process. Any amount of search will scan a variety of different available units, not randomly, but in accordance with a selection procedure that has only inadequate information: hence the need for closer, often face-to-face confrontation. The greater the number of units examined the closer to the ideal is likely to be the best unit in the sample examined. Thus, we can measure the amount of search in terms of the expected degree of potential improvement discovered--i.e., the expected decrease in L^* by moving to the most appropriate unit discovered. So the search (or information) cost function also expressed in present discounted value, is as follows:

$$(17) \quad C_I^* = C_I^* (\hat{\Delta} L^* / L_t^*)$$

where $\hat{\Delta} L^*$ is the expected decrease in disequilibrium cost from L_t^* due to more suitable housing being discovered by market search;

L^* is the original disequilibrium cost.

Then total market participation cost is:

$$(18) \quad C^* = C_T^*(K^i, r^*) + C_I^* (\hat{\Delta} L^* / L_t^*)$$

$$\partial C_T^* / \partial K^i < 0, \quad \partial C_T^* / \partial r^* > 0, \quad dC_I^* / dDL > 0, \quad d^2 C_I^* / dDL^2 > 0$$

where $DL = \hat{\Delta} L^* / L_t^*$.

Household i will enter the market as an active participant if the expected decrease in L^* exceeds the expected cost of that participation. The conditions for an optimal

decision are:

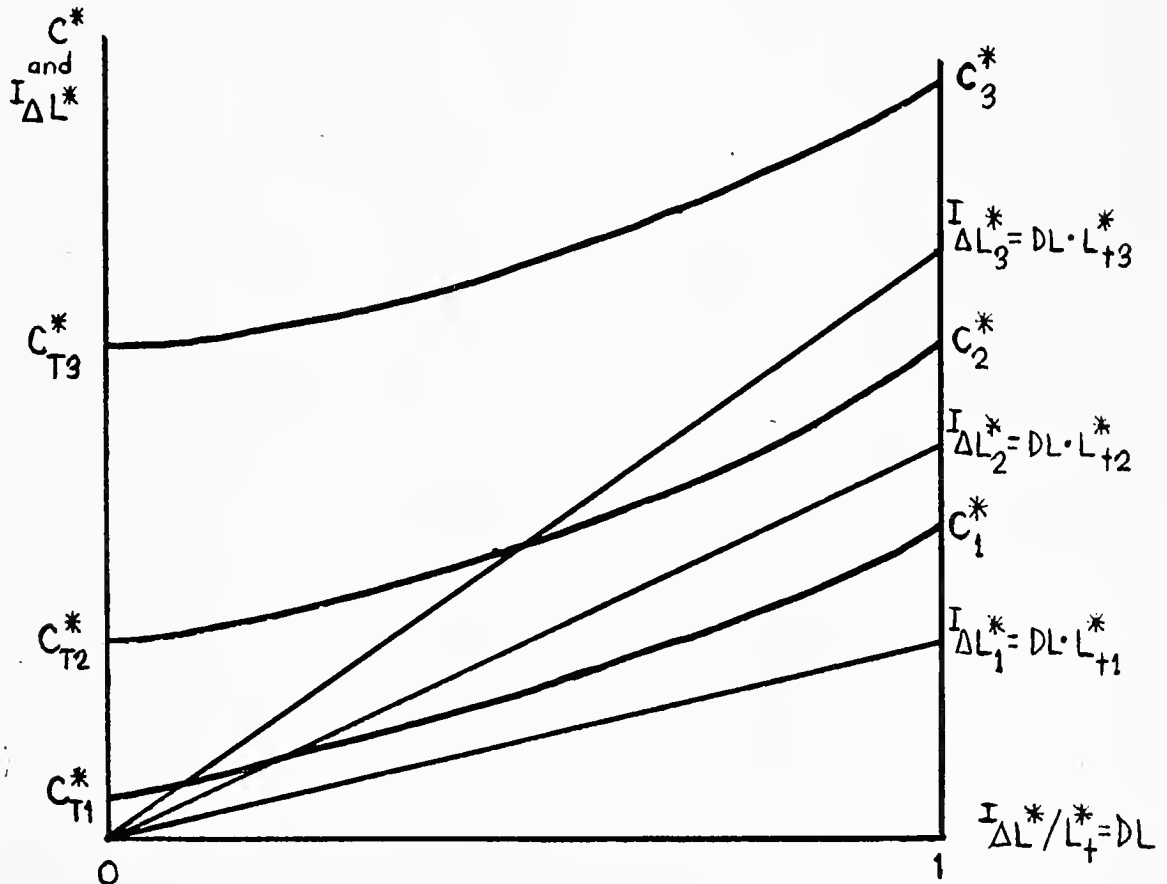
$$(19a) \quad \Pi(DL) = (DL)L_t^* - C^*(DL, K^i, r^*) > 0 \quad \text{for some } DL$$

$$(19b) \quad d\Pi/dDL = 0, \quad d^2\Pi/dDL^2 < 0 \quad \text{within the range where } \Pi > 0$$

Equation (19a) establishes the net gain of at least one possible degree of active participation. (19b) determines the optimal amount of participation, in terms of the extent of search, as where expected marginal gain from additional search is just offset by the marginal cost of that extra search.

Figure 2 shows the dependence of the optimal action on K^i , r^* , and L_t^* :

Figure 2



- a. For a very low initial disequilibrium cost L^*_{t1} no expected amount of search is worthwhile -- all the cost curves exceed ΔL^*_1 everywhere. For higher L^* there are ranges of expected search (DL) where $\Delta L^*_1 > C^*$ (except where C^*_t is very high -- e.g. C^*_{T3}). In effect, $\partial^2 C^* / \partial \Delta L^* \partial L^*_t < 0$, i.e. high L^*_t lowers marginal search cost of unit improvement.
- b. For high K^i or low r^* , C^*_t is generally low enough to make some expected search levels profitable (e.g. C^*_1, C^*_2); where the K^i, r^* combination is such that $C^*_t = C^*_{T3}$, expected costs exceed expected gains even where there is a very high initial disequilibrium.
- c. Since market conditions gradually diverge more and more over time from what they were at some initial point (when household i moved to its current residence), and household i's own demographic circumstances are likely to diverge cumulatively over time from that same initial period, the size of the housing disequilibrium is likely to grow over time -- $\dot{L}^* > 0$ generally. As a result, an initial disequilibrium at t, L^*_t , insufficient to warrant active market participation, will gradually increase over time until some market transaction is prospectively profitable.

Assuming household i becomes an active participant in the market, its equilibrium transaction will occur where

$$(20) \quad \partial \Delta L^* / \partial DL = \partial C^* / \partial DL$$

i.e., the marginal expected reduction in loss equals the marginal cost of bringing about that expected reduction. This equilibrium generally occurs at an expected reduction less than the original loss ($\Delta \hat{L}^* / L^*_t < 1$). Moreover, the larger is L^*_t the larger will be $\Delta \hat{L}^* / L^*_t$. But the absolute relationship is not clear: $\partial \hat{L}^* / \partial L^*_t \leq 0$.

B. Aggregate Short-Run Demand

1. Aggregation in Terms of Housing Packages

At any given time each household formulates an optimal housing package, relates this to its actual housing, and decides whether the discrepancy between the two is great enough to warrant it going into the market. If it is, the household will be prepared to carry out a degree of search necessary to decrease its loss from the disequilibrium by a designated expected amount through approximating its ideal target. Since the outcome of an actual course of search will diverge from \hat{H} in only a random fashion, the expected configuration of search termination can be given as equal to \hat{H} . The variance of the actual best H found will be a positive function of $L_t^* - \hat{L}^*$.

Thus, when aggregating the demand of the whole city population, each household can be treated as demanding housing as follows:

- a) If inactive in the market, household i demands one unit identical with its present occupancy -- i.e., H_t
- b) If active in the market, household i demands one unit equal to its calculated optimal package -- i.e., \hat{H} .

So we obtain aggregate demand simply by enumerating the number of units of each type demanded by the population as a whole. Symbolically:

$$(21) D_H = D \left[(F^i), (H^i), P_1, \dots, P_M \right] \quad \text{all } H$$

where D_H is the number of units of type H demanded; (F^i) is the set of demographic profiles F^i of the city population; P_1, \dots, P_M are the housing component prices.

At any time t , all three sets of variables will be available, so the demand for each kind of dwelling can be determined.

2. Housing Quality

As we argued above, (H) establishes little more than a nominal scale. It is extremely

awkward to attempt to understand market forces in terms of multi-dimensional packages. We seek some way to increase the comparability of different housing packages so as to be able to understand the working of the market at a more general -- and powerful-- level.

We argued in section II of this essay that heterogeneity makes for a segmentation of the market into virtual submarkets, with various degrees of substitutability among submarkets. The submarket, then, is the proper level at which to aggregate demand and supply forces, and at which adjustments and repercussions should be examined.

The problem is how to aggregate the individual units into these submarkets, so that:

1) the groupings will conform to market aggregates that really do reflect a functional homogeneity in market operations; 2) the relationships between and among groupings can be compactly and efficiently understood and expressed.

The clusterings of close substitutes, and the patterns of graduations of substitutability among units and clusters cannot confidently be asserted to be perfect in the sense of metric scalability (so that everybody agrees on the degree of substitutability between any two units, and degree of substitution forms a perfectly transitive ordering). But this problem is not unique with housing. Most commodities show aberrations when closely studied. Even so conventional and seemingly assured classifications as "industries" exhibit serious imperfections when actual market substitution is involved.⁵ Nonetheless we make use of classifications like commodity and industry for most analytic and statistical work in economics. We shall assume that in terms of actual

⁵Anti-trust court proceedings on the aluminum industry is one of several which showed significant slippage in conventional classifications of industries when real market behavior was studied.

market processes the units of the standing housing stock can be tolerably arranged in terms of degree of substitutability (referring to disagreements about the position of any unit in the array) and moreover, that this array is approximately linear (i.e., that degree of substitutability is a transitive relationship).

Insofar as the real world obeys these assumptions, and that we can make actual assignments to this array, we possess a strong analytic instrument for understanding the complex of market activities. Because we shall know the whole structure of market repercussions resulting from a change originating in any part of the system. Each change will radiate from the point of origin typical substitutive types of adjustment which weaken monotonically with substitutional distance from that origin. Given the salient multidimensionality of housing, this enables me to avoid the highly misleading treatment of housing as a homogeneous commodity which varies only by quantity, while avoiding the opposite pitfall of having to deal with a market of thousands of different kinds of commodities whose relations with one another are idiosyncratic and require pair-by-pair elucidation.

Thus, we assume an orderly abstract space of substitutional distance (degree of substitutability) along a single organizing dimension. This organizing dimension we label "housing quality." A number of different units, units differing from one another in various dimensions of the housing package, will typically fall at each level of housing quality. Thus, in effect the multi-dimensionality of the housing package is being collapsed for this purpose to a single dimension. How is this mapping from M-space to a single dimension accomplished?

The utility function of each household makes utility level a positive function of the housing components. So for each household, utility differences mark degrees of substitutability within any set of housing units. But household preferences differ, so

there is no consensus among the population with respect to a direct array of units along the quality scale.

An alternative would be for some external source--e.g. a group of experts of housing--to judge the relative quality of different houses. This leaves much to be desired, since "quality" is not an "objective" characteristic observable by inspection. It is an inferred property, stemming from actual market transactions. It is the market that must indicate the appropriate mapping.

The market can perform the task, even in the face of considerable differences in tastes, in the following way. At each equilibrium position of the market, every consumer has adjusted his budgetary allocation so that the marginal rate of substitution between any pair of commodities equals the price ratio between them. In highly competitive markets the prices facing all consumers are equal. So at equilibrium all consumers have the same marginal rate of substitution between the pair. This equality holds for sellers (producers) also. Their marginal rate of transformation (or ratio of marginal costs) is equated to the common price ratio--and thus, to the common marginal rate of substitution. Thus, a common tradeoff exists, and is measured by the observable price ratio of consumers. This is an important form of consensus.

In the case of housing what we require in order to apply this approach is that the several housing components be subject to substitution by households. Households do in fact show a willingness to substitute one component for another. There are links, because within some ranges components are complementary with one another, rather than substitutive (few families would want a unit with twenty bedrooms and only 1 bathroom). But the ranges where components are treated as substitutes (land for interior space, accessibility for land or interior space, neighborhood amenities for accessibility,

interior space for condition, etc.) are substantial. So preference tradeoffs among the components can be established and we can make use of the market equilibrium consensus or marginal tradeoffs to help construct an index of housing quality.

Assume that all households, despite differences in taste, have the same functional form for the effect of housing on utility. Only the parameter values differ. We can construct a representative function which relates housing components to "representative gratification" levels by using the common ratios of marginal utility discoverable at market equilibria. Expressing each marginal rate of substitution as that between a housing component and the numeraire commodity Z , we obtain:

$$(22) \quad U_{Hi} = P_{Hi} \quad \text{all } i$$

where U_{Hi} is the partial derivative of the "representative gratification" function;

P_{Hi} is the implicit price of housing component i

Since we define the consensus as that which exists in the particular equilibrium, each U_{Hi} is a constant, not a variable. Thus, the "representative gratification" function is linear. We define it as follows:

$$(23) \quad G = \sum_{i=1}^M a_i H_i$$

This is, of course, a form of hedonic index. From (23), we solve for a_1, a_2, \dots, a_M :

$$(24) \quad a_i = P_i$$

We can use this function as an index of housing quality. Each unit will have a G value corresponding to its composition of components. Then the standing stock can be arrayed by G value as a distribution of quality levels, and we thereby approximate the degree of substitutability distribution in the market.

It is important to realize that our G function does not measure individual utility,

because it is based on constant marginal utilities, and thus constant marginal tradeoffs across components. The marginal tradeoffs for any household are likely to change for different patterns of consumption. Thus, when faced with a set of housing units all possessing the same G value but consisting of quite different internal compositions, a household will have definite preferences among them, because they will in fact give different utility. So housing quality and utility are different. The closer is the composition of the package to the one actually purchased at the base equilibrium, the closer will its utility be to that purchased at equilibrium.

G as a measure of housing quality should be compared to the h^i variables discussed above--since the latter also measure "amount" or quality of housing. They are in fact different, because each h^i index represents an evaluation of housing level from the point of view of household i 's complete set of marginal preferences, not just a linear projection from his marginal preferences at the base equilibrium.

How useful can this index of housing quality be? Since it represents a projection from one aspect of the base equilibrium situation, it is most valid where that aspect of the base situation persists. But this does not mean that all aspects of that situation must persist. What is important is the set of relative prices of housing components -- not the other aspects. These relative prices may remain relatively unchanged even through a succession of considerable changes in the market --equilibrium and disequilibrium periods. Just as in production theory we suppose the business firm may experience a variety of output level changes while assuming that optimal input combinations remain unchanged.

Constant relative component prices may persist over time if housing materials are supplied by approximately constant cost industries who face relatively similar inflationary

pressures over time. It is not necessary that all building materials, for example, experience similar price histories, since disproportionate rises in some will often lead to heavy substitution away from them to alternative materials. Urban land is not necessarily an exception to this. Urban land can be created, just as building materials. It occurs when land hitherto used for non-urban purposes becomes converted to urban use. With essentially the same population, modest changes in overall land for urban use can be brought about at little extra cost. Substantial changes would, however, be likely to raise land prices relative to that of other components, unless the land expansion accompanied a general expansionary period in which building materials prices rose correspondingly.

The above is not meant to argue that an initial configuration of relative component prices will persist over all kinds of circumstances, but only that it is by no means restricted to the initial equilibrium situation alone. A variety of changes overtime are compatible with the continued tolerable validity of a given G function. Of course, more salient changes requires reestimation of a G function.

3. Aggregation in Terms of Housing Quality Levels

In order to integrate the concept of housing quality with our previous discussion of consumer equilibrium, we introduce an alternative form of the household utility function:

$$(25) \quad W^i = W^i [Z, G, (R)]$$

where W^i is household i 's utility level;

G is the housing quality level;

(R) is a set of variables describing the relative composition of the housing package.

To use this in formulating the household's equilibrium, we need to define prices for the different quality levels. This is an appropriate requirement because we have argued that the segmentation of the market means that each cluster of closely substitutable units has a degree of independent variation from that of clusters imperfectly substitutable with it. Each cluster is in effect a separate commodity, and while its price cannot go far out of line with close substitutes, it can with very weak substitutes. So each quality level can have a separate price which does not simply represent differences in quality level itself.

From (23) and (24) we can write for G :

$$(26) \quad G = \sum_{i=1}^M P_i H_i$$

It is quite reasonable to suppose that market prices of housing packages approximate an adding up of the prices of the constituent components.

Then (26) suggests that G is an expected market value of the housing package--i.e. expected on grounds of the value of its components. But differing degrees of market tightness in each of the quality level clusters may drive actual market price for a unit in each cluster above or below this. So a measure of the price of buying at some quality level j is:

$$(27) \quad p_j = MV_j / G_j$$

where MV_j is the actual market value of a unit at quality level j .

Household i 's equilibrium may now be looked at as follows:

$$(28) \quad \max W^i [Z, G, (R)]$$

subject to the budget constraint

$$Y^i = Z + MV(G) G$$

where $MV(G) G$ is the total expenditure on the selected unit at level G .

Given P_1, P_2, \dots, P_M and $p_1, p_2, \dots, p_i, \dots$, household i will select an optimal $[\hat{G}, (\hat{R})] = \hat{H}$. So the earlier choice of \hat{H} is expressed here in terms of quality level and composition.

We now aggregate this demand in terms of quality levels:

$$(29) \quad D_G = \bar{D} [(F^i), (H^i), P_1, \dots, P_M, (p_1, \dots, p_i, \dots)] \text{ all } G$$

where D_G is the number of units demanded at quality level G .

This formulation enables one to calculate the structure of demand at any time in terms of the different quality submarkets. It thus permits understanding of the distribution of demand by closeness of substitutability and thus facilitates explanation of the consequences of different impulses stemming from the supply side of the market. Only after such interactive processes are formulated can the distinctive working of the housing market complex and the consequences of different public policies be understood. Part II of this essay will deal with the supply and market adjustment aspects, and Part III with public policy.

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